Mathematical modelling and stability analysis of Alcoholism with Simulink

PROJECT REPORT

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of MATH F420

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Introduction

Epidemiological Models date back to the late 1920s where Kermack and McKendrick used such models to study the plague. From there, the various generalizations of epidemic models have enabled us to represent most epidemics with a great deal of accuracy. Drawing inspiration from this, We use the common SATQ model [1] to model alcoholism as a compartmental model. The rest of this report is organized as follows: We first introduce the model, discuss the reproduction number, give the results of stability at the alcoholism free equilibrium point and then we give the graphs.

1 Model

Here in our Model S represents the susceptible population. A represents the alcoholics, T represents the population of people whi have accepted they have an issue and are in treatment and Q represents those people who have quit altogether.

$S' = bN - \beta\frac{SA}{N} - dS$

$A' = \beta\frac{SA}{N} - \xi T - (d + 0.5)A$

$T' = 0.5A - (d + \xi + \delta)T$

$Q' = \delta T - dQ$

$N' = (b - d)N$

b	Natural Birth Rate
d	Natural Death Rate
β	Rate from class S to A
ϵ	Rate from A to T
δ	Rate from T to Q

2 Reproduction Number

In order to find the reproduction number we reformulate the state space model of the SATQ model as follows: Let $X = (A, T, Q, S)$. Then we can write the state space model as:

$$\frac{dX}{dt} = \mathcal{F}(x) - \mathcal{V}(x)$$

where we define $\mathcal{F}(x)$ and $\mathcal{V}(x)$ as follows

$\mathcal{F}(x) = \begin{pmatrix} \beta\frac{SA}{N} \\ 0 \\ 0 \\ 0 \end{pmatrix}$ and

$\mathcal{V}(x) = \begin{pmatrix} (0.5 + d)A - \xi T \\ (d + \xi + \delta)T - 0.5A \\ dQ - \delta T \\ \beta\frac{SA}{N} + dS - bN \end{pmatrix}$ and

$\mathcal{DF}(Eo) = \begin{pmatrix} \beta\frac{b}{d} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ and

$\mathcal{DV}(Eo) = \begin{pmatrix} 0.5 + d & \xi & 0 & 0 \\ 0.5 & d + \xi + \delta & 0 & 0 \\ 0 & -\delta & d & 0 \\ \beta & 0 & 0 & 0 \end{pmatrix}$ and

Finally we give the result for the reproduction number as follows:

$$R_0 = \rho(FV^{-1}) = \frac{b\beta(d+\xi+\delta)}{0.5d(d+\delta+d(d+\xi+\delta)}$$

3 Stability Analysis

3.1 Stability at Alcoholism Free Equilibrium

$$\mathcal{J}(Eo) = \begin{pmatrix} \beta(d+0.5) & \xi & 0 & 0 \\ 0.5 & -(d+\xi+\delta) & 0 & 0 \\ 0 & \delta & -d & 0 \\ -\beta & 0 & 0 & -d \end{pmatrix}$$

We can easily solve for two eigenvalues $\lambda_1 = \lambda_2 = -d < 0$, while λ_3 and λ_4 satisfy the equation
$\lambda^2 + [2d+\xi+\delta+0.5-\beta]\lambda + (d+\xi+\delta)(d+0.5-\beta) - 0.5\xi = 0$

Manipulation of this equation proves that Real parts of both λ_3 and λ_4 are less than 0.
Re $\lambda_3 < 0$, Re $\lambda_4 < 0$

3.2 Existence of endemic equilibrium Point

On solving the four equations $S' = 0$, $A' = 0$, $T' = 0$ and $Q' = 0$ simultaneously we get the following equilibrium point.

$S^* = \frac{0.5bN^2}{0.5dN+T^*\beta(d+\xi+\delta)}$

$A^* = \frac{(d+\xi+\delta)T^*}{0.5}$

$Q^* = \frac{T^*\delta}{d}$

$T^* = \frac{\sigma}{\beta(d+\xi+\delta)[d\xi+(\delta+d)(d+0.5)]}$
where,
$\sigma = 0.5bN\beta(d+\xi+\delta) + 0.5^2dN\xi - 0.5dN(d+0.5)(d+\xi+\delta)$

Given all the quantities this shows the existence of an endemic equilibrium point

3.3 Numerical Simulation using Simulink

For the case of numerically solving our SATQ model, we used the following constants:

b	0.250025
d	0.25
β	0.55
ϵ	0.8
δ	0.3
N_0	1000
S_0	650
A_0	350
T_0	80
Q_0	20

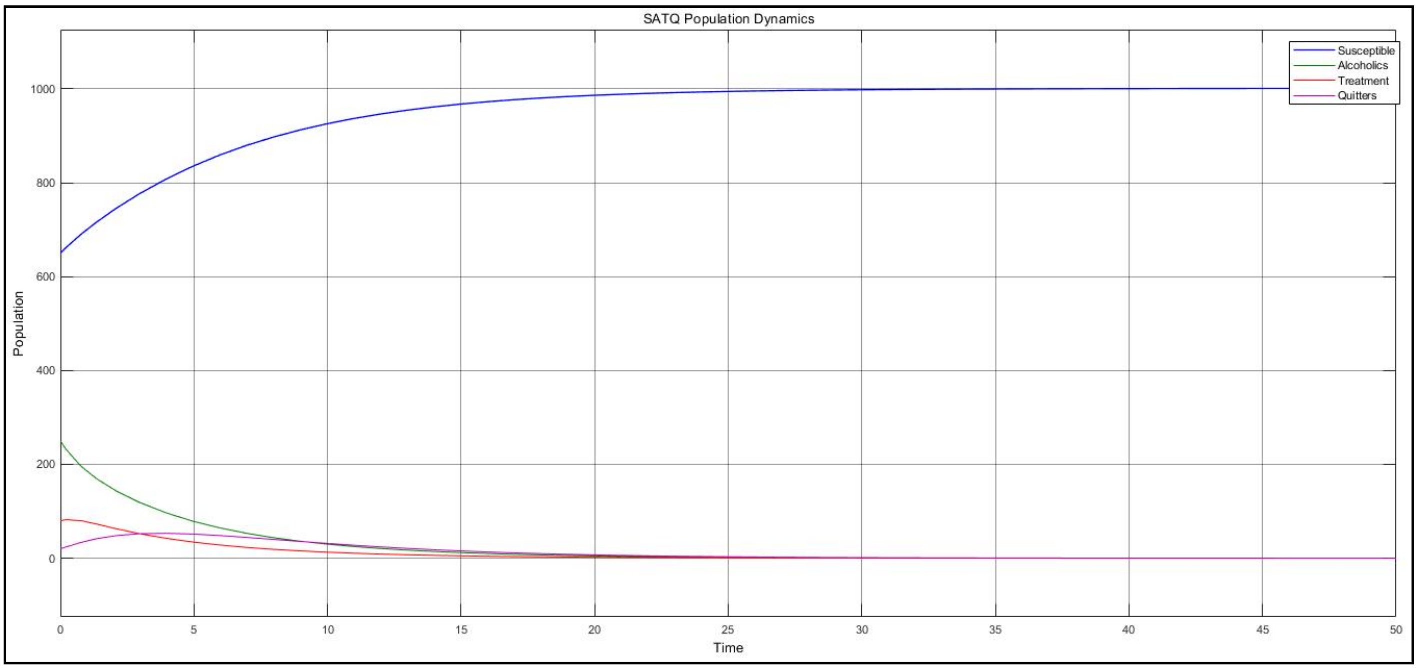

Figure 1: SATQ Dynamics

4 Model in terms of proportions

We also transform the model using the transform $sN = S$, $aN = A$, $tN = T$ and $qN = Q$. This transformation can be thought off as a conversion from absolute numbers of Susceptible, Exposed, infected and Recovered populations to that of proportions.
Now, it is important to note that on using this transformation we can't simply write $S' = Ns'$. Working with a non-constant population, we must include its effect in this transformation.
Therefore, it is correct to write $S' = s'N + sN' = s'N + (b-d)N$. Similar results follow for A,T,Q.

The transformed model is as follows:

$$s' = b - \beta sa - bs$$

$$a' = \beta sa - \xi t - (b + 0.5)a$$

$$t' = 0.5a - (b + \xi + \delta)t$$

$$q' = \delta t - bQ$$

4.1 Numerical Simulation using Simulink

Using the same constants as before we simply plot the SATQ dynamics for the model in terms of proportions:

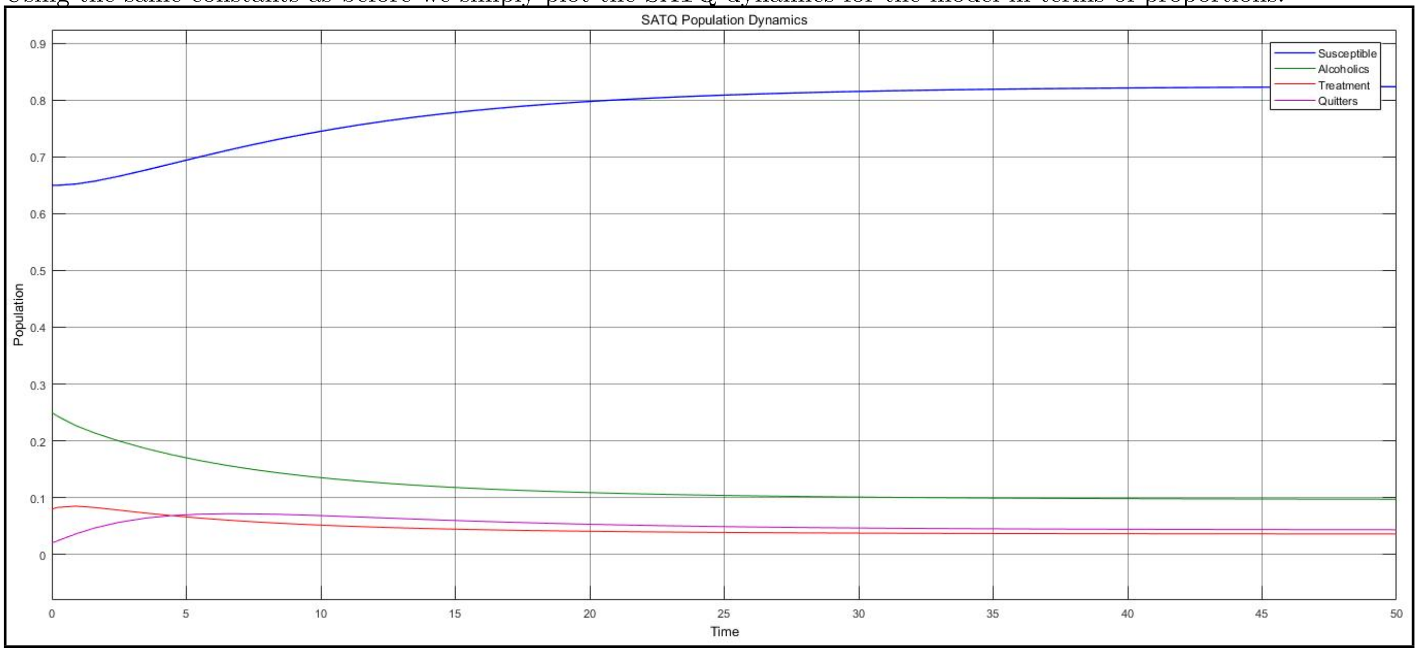

Figure 2: satq Dynamics

REFERENCES

[1] Xun-Yang Wang, Hai-Feng Huo, Qing-Kai Kong, and Wei-Xuan Shi, "Optimal Control Strategies in an Alcoholism Model," Abstract and Applied Analysis, vol. 2014, Article ID 954069, 18 pages, 2014. https://doi.org/10.1155/2014/954069.